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| **UNIT-III - (Bivariate Distribution)**  Definition of Bivariate distribution and their properties, Distribution of sums and quotient, conditional densities |
| **Joint Distribution**  Here we consider the typical case of two random variables which are either both discrete or both continuous.  **Discrete Case:**  If X and Y are two discrete random variables we define the joint probability function of X and Y by  P (X= x, Y= y) = f(x,y)  Where, 1.  2.  A joint probability density function for X and Y can be represented by a joint probability table:   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | y1 | y2 | ... | yn | Total | | x1 | f(x1,y1) | f(x1,y1) | ... | f(x1,y1) | f1(x1) | | x2 | f(x1,y1) | f(x1,y1) | ... | f(x1,y1) | f1(x2) | | .  . | .  . | .  . | .  . | .  . |  | | xm | f(x1,y1) | f(x1,y1) | ... | f(x1,y1) | f1(xm) | |  | f2(y1) | f2(y2) |  | f2(yn) | 1 |     Grand Total  **Continuous Case:**  The joint probability density function of random variable X and Y is defined by       The probability that X lies between a and b while Y lies between c and d is given by |
| **Marginal Density and Distribution Function**  **For Discrete RV:**   1. **The marginal density function of X = xj is given by**     **And the marginal density of Y = yk is given by**     1. **The Joint Distribution Function of X and Y is defined by**      1. **The marginal distribution of X and the marginal distribution of Y is**   and  **Note:**   1. The r. v.’s (X1, X2, X3,....Xn) are independent if and only if their joint p.m.f is equal to the product of their marginal p.m.f’s. i.e.   **For Continuous r.v:**   1. **The marginal density function of X and the marginal density function of Y is**   **and**   1. **The marginal distribution function of X and Yare given by**     **Note:**   1. X and Y are independent random variables if     Or equivalently |
| **Distribution of Sum and Quotient**  **Note:** The density function of the sum of two continuous random variables X and Y of  U = X+Y, having joint density function f (x,y) is given by    **Theorem 1:** If X and Y are independent continuous r. v.’s then the p.d.f of U=X+Y is given  by :  **Theorem 2**: If X and Y are independent continuous r. v.’s then the p.d.f of U=X - Y is given  by :  **Theorem 3**: If X and Y be two independent continuous r. v.’s then the p.d.f of their product  U = XY is given by:    **Theorem 4**: If X and Y are independent continuous r. v.’s then the p.d.f of the quotient  Z=X/Y is given by: |
| **Conditional Distribution Function and Conditional Probability Density Function**  For two dimensional random variable (X, Y), the joint distribution function FXY(x,y) for  any real numbers x and y is given by:    The conditional distribution function FY/X(y/x) denotes the distribution function of Y  when X has already assumed the particular value x. Hence    Using this, the joint distribution function FXY(x, y) may be expressed in terms of the  conditional distribution function as follows:    **Similarly**  The **conditional probability density function** of Y given X for two random variable X and Y which are jointly continuously distributed is defined as follows, for two real numbers x and y:    **Note:**   1. Conditional prob of Y given X -where*, f1(x)* is the marginal probability for X. 2. Conditional prob of X given Y-, where *f2(y)* is the marginal probability for Y 3. Probability of Y between c and d given that x < Y < x + dx is |
| **CLASSWORK PROBLEMS**  **(Joint Probability Distribution, marginal Distribution Conditional density)**   |  |  |  |  | | --- | --- | --- | --- | | **Q1** | The joint probability function of two discrete random variables X and Y is given by , where *x* and *y* can assume all integers such that  and *f(x,y) = 0*, otherwise  Find (a) the value of the constant *c*  (b)  (c) *P(X = 2, Y = 1)*  (d) Marginal probability function of X and Y. | |  | |  | Sol**: *(a) c = 1/42, (b) 5/42, (c) 4/7*** | |  | | **Q2** | The joint density function of two continuous random variables X and Y is    Find (a) the value of the constant *c* (b) P (1 < X < 2, 2 < Y < 3)  (c)  (d) Marginal distribution function of X and Y | |  | |  | Sol: ***(a) c=1/96 (b) 5/128 (c) 7/128*** | |  | | **Q3** | The joint probability distribution of two random variables X and Y is given by:  P (X = 0, Y = 1) = 1/3, P(X = 1, Y = -1) = 1/3, P( X = 1, Y = 1) = 1/3,  Find (a) Marginal distribution of X and Y (b) the conditional probability distribution of X given Y = 1. | |  | |  | Sol: **(*a) x: -1 0 1 & y: -1 0 1***  ***P(x): 0 1/3 2/3 P(y): 1/3 0 2/3***  ***(b): Conditional Distribution: x: -1 0 1***  ***P(X= x | Y=1) : 0 1/2 ½*** | |  | | **Q4** | The joint probability density function of a two dimensional random variable (X, Y) is given by:   1. Find the marginal density function of X and Y. 2. Find the conditional density function of Y given X = x and conditional density function of X given Y = y. 3. Check for independence of X and Y | |  | |  | ***Sol: (a)***   1. ***1/x, 1/(1-y) (c) not independent*** | |  | | **Q5** | | Joint distribution of X and Y is given by :  Test wheather X and Y are independent. For the above joint distribution, find the conditional density of X given Y = y | |  | | ***Sol: independent,*** | | **Q6** | | Let X and Y be jointly distributed with p.d.f.:  Show that X and Y are not independent but X2 and Y2 are independent. | | **Q7** | | The joint density function of the random variables X and Y is given by:    Find (a) the marginal density of X, (b) the marginal density of Y (c) the conditional  density of X (d) the conditional density of Y. | |  | | ***Sol:  (c)***  ***(d)*** | | **Q8** | | If X and Y are independent r.v’s having density functions  , Find the density function of their sum, U = X + Y | |  | | ***Sol:*** |     **TUTORIAL QUESTIONS**   |  |  |  | | --- | --- | --- | | **Q 1** | The joint probability function of two discrete random variables X and Y is given by , where *x* =1, 2, 3 and *y* = 1, 2, 3 and *f(x, y) = 0*, otherwise  Find (a) the value of the constant *c* (b)  (c) P*(X = 2, Y = 3)*  (d)  (e) P(Y < 2) (f) P(X = 1) (g) P( Y = 3)  (d) Marginal probability function of X and Y. |  | |  | Sol: ***c = 1/36, 1/6, 1/4 , 5/6, 1/6, 1/6, 1/2*** |  | | **Q 2** | Let X and Y be continuous r.v. having joint density function    Determine (a) the constant c (b) (c)  (d)  (e) whether X and Y are independent. |  | |  | ***Sol: 3/2, 1/4, 29/64, 5/16, Dependent*** |  | | **Q 3** | Suppose that two dimensional continuous random variable (X, Y) has joint p.d.f given by :   1. Verify that  (b) Find P(0 < X < 3/4, 1/3 < Y < 2), (c) P (X + Y < 1), (d) P ( X< 1| Y < 2) |  | |  | ***Sol: (b) 3/8, 1/10, 3/5, 1*** |  | | **Q 4** | Given: . Are X and Y independent?  Find (a) P(X > 1), (b) P (X < Y|X < 2Y), (c) P (1 < X+Y < 2) |  | |  | ***Sol: independent (a) 1/e (b) 3/4 (c) 2/e - 3/e2*** |  | | **Q5** | If X and Y have the joint density function    Find (a) f (y | x) (b) P ( Y> ½ | ½ <X <1/2 +dx) |  | |  | ***Sol: (a)*** ***(b) 9/16*** |  | | | |
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